Algebraic–Differential Cryptanalysis of DES

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Journées C2
Plan

1 Introduction

2 Algebraic cryptanalysis of DES using Minisat
   - Data Encryption Standard
   - Modeling
   - Experimental results

3 Algebraic–differential cryptanalysis of DES
   - Algebraic–differential cryptanalysis
   - Results on six, seven and eight rounds
1 Introduction

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Algebraic Cryptanalysis

Claude Shannon:

"Breaking a good cipher should require as much work as solving a system of simultaneous equations in a large number of unknowns"
Algebraic Cryptanalysis

- **Algebraic representation** of a cryptographic primitive.
- Tools for efficient **polynomial system solving**.
  1. Gröbner Bases algorithms (Buchberger, Faugère F4 and F5).
  2. SAT Solvers.

**Remark**

There is a very strong link between the modeling and the tools used for the resolution.

**Challenge**

Can algebraic cryptanalysis be efficient against block ciphers?
Our work

- SAT Solvers attacks against DES using different modelings of the DES S-boxes.
- Incorporation of elements from differential cryptanalysis
  - new attacks against 6, 7 and 8 rounds of DES using dedicated characteristics.
- Tradeoff between time and data complexity.
SAT Solvers?

- Very efficient and flexible dedicated softwares.
- SAT-competition. Active research field.
- Easy to use. Low memory consumption.
Courtois N.T., Bard G.V. and Jefferson C.

Efficient Methods for Conversion and Solution of Sparse Systems of Low-Degree Multivariate Polynomials over GF (2) via SAT-Solvers.


- Replace each monomial by a new variable.
- Cut linear equations into smaller equations (by adding new variables).
- + optimizations.

MiniSat2

Een, N. and Sorenssson, N.

MiniSat: A SAT solver with conflict-clause minimization
Sparse quadratic systems

(time in seconds)

monomials/polynomial
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Data Encryption Standard

- Iterative Block Cipher
- Block size: 64 bits
- Effective size of the key: 56 bits
- Encryption standard between 1976 and 2002

Why did we choose to study the DES?
Main attacks against DES

Wiener, M.J.
*Efficient DES key search.* Technical Report

Biham, E. and Shamir, A.
*Differential cryptanalysis of the full 16-round DES.* Crypto’1992

Knudsen, L.R.
*Partial and higher order differentials and applications to the DES.* BRICS report

Matsui, M.
*Linear cryptanalysis method for DES cipher.* EUROCRYPT’1993
**DES Structure**

**Feistel network**

Diagram:
- **Plaintext (64 bits)**
- **IP**
- **F’**
- **for 16 rounds**
- **FP**
- **Ciphertext (64 bits)**
DES Structure

Feistel network

S-boxes: non-linear part of the system
Algebraic cryptanalysis of DES using Minisat

Starting point

N.T. Courtois, G.V. Bard
*Algebraic Cryptanalysis of the Data Encryption Standard*
IMA Int. Conf. 2007

General principle

- 1 known plaintext.
- Model the cryptosystem by a set of clauses.
- Use Minisat to extract the key.

Remark

We can combine this approach with an exhaustive search over some bits of the key.
S-boxes modeling (I)

We have considered several modelings of the DES S-boxes.

The choice of the modeling is very important.

Our modeling

We search (exhaustively) for the set of polynomials which verify:

\[ P(x_1, \ldots, x_6, y_1, \ldots, y_4) = \prod (x_i + \alpha_i) \prod (y_i + \beta_i), \alpha_i, \beta_i \in \{0, 1\} \]

such that

\[ S(x_1, \ldots, x_6) = (y_1, \ldots, y_4) \Rightarrow P(x_1, \ldots, x_6, y_1, \ldots, y_4) = 0 \]

Complexity : $3^{10}$
### S-box modeling (II)

<table>
<thead>
<tr>
<th>S-box</th>
<th>Nb of clauses</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>1624</td>
</tr>
<tr>
<td>S2</td>
<td>1844</td>
</tr>
<tr>
<td>S3</td>
<td>1767</td>
</tr>
<tr>
<td>S4</td>
<td>1881</td>
</tr>
<tr>
<td>S5</td>
<td>1812</td>
</tr>
<tr>
<td>S6</td>
<td>1705</td>
</tr>
<tr>
<td>S7</td>
<td>1673</td>
</tr>
<tr>
<td>S8</td>
<td>2047</td>
</tr>
</tbody>
</table>

For 6 rounds: 792 variables and 90086 clauses. + partial exhaustive search on 28 bits of the key.
Experimental results
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   - Algebraic–differential cryptanalysis
   - Results on six, seven and eight rounds
Our approach

Limit

Algebraic cryptanalysis usually consider only one known plaintext.

We combine algebraic cryptanalysis and statistical techniques to exploit efficiently the knowledge of several plaintexts.

- **Tradeoff** time/plaintexts.
- In particular, we consider differential cryptanalysis.
Differential cryptanalysis (I)

- The principle was known by the DES designers.
- Based on a statistical bias of the S-boxes.
- Key recovery attack.
- We try to predict how the difference of a pair of plaintexts will diffuse through the cipher.
Differential cryptanalysis of the 3 round-reduced DES

\[
\Delta_{\text{in}} = B = A_{3L}
\]

\[
\Delta_{\text{out}} = A_{0L} \oplus A_{3R}
\]

\[
A_{0L} + 00 00 00 00x \rightarrow F_1 \rightarrow 00 00 00 00x
\]

\[
B + 00 00 00 00x \rightarrow F_2 \rightarrow A_{0L}
\]

\[
A_{0L} + B \rightarrow F_3 \rightarrow A_{3L} \oplus A_{3R}
\]
Differential cryptanalysis (II)

more than 3 rounds

- Statistical method.
- *differential characteristics.*
- A lot of plaintexts needed.
Motivations

Compromise between algebraic cryptanalysis and differential cryptanalysis.

- We can use the strong correlation between the subkeys.
- The notion of difference is easy to represent with clauses.
- We only need one pair following the characteristic to retrieve the key.
General algorithm

Repeat until the key is found:

- Choose a differential characteristic.
- Choose two plaintexts with difference fixed by the characteristic.
- Construct the system of clauses for DES, and add the clauses corresponding to the characteristic.
- Solve with MiniSat. If the result is UNSATISFIABLE, restart (it means that the pair didn’t follow the characteristic). If the result is SATISFIABLE, then MiniSat returns the key.
Six rounds

Approach

- Classical differential characteristics.
- **Combination** of different characteristics to reduce the data complexity.
Six rounds

**Approach**

- Classical differential characteristics.
- **Combination** of different characteristics to reduce the data complexity.

**How to combine**

- We can run MiniSat 6 times with 7 plaintexts.
- Six 3-rounds characteristics $\Delta_1, \ldots, \Delta_6$.
- 7 plaintexts $m_0, \ldots, m_6$ such that $m_i = m_0 \oplus \delta_i$. 
## Experimental results

<table>
<thead>
<tr>
<th>Cryptanalysis</th>
<th>Plaintexts</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Differential (Biham, Shamir)</td>
<td>240</td>
<td>0.3 seconds</td>
</tr>
<tr>
<td>Differential (Knudsen)</td>
<td>46</td>
<td>a few seconds</td>
</tr>
<tr>
<td>Algebraic with SAT Solver (Courtois, Bard)</td>
<td>1</td>
<td>$2^{25}$ seconds</td>
</tr>
<tr>
<td>Algebraic-differential</td>
<td>32</td>
<td>3000 seconds</td>
</tr>
<tr>
<td>Algebraic-differential (combination of characteristics)</td>
<td>22</td>
<td>&lt;10 hours</td>
</tr>
</tbody>
</table>
Seven rounds

For seven rounds and more, the classical differential characteristics don’t seem to be adapted.

We have used a dedicated differential characteristic.

- Truncated characteristic with probability $1/1000$. 
### S-Box

<table>
<thead>
<tr>
<th>S-Box</th>
<th>$\delta_{in}$</th>
<th>$\delta_{out}$</th>
<th>Proba</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>$4_x$</td>
<td>$6_x$</td>
<td>10/64</td>
</tr>
<tr>
<td></td>
<td>$8_x$</td>
<td>$3_x$</td>
<td>12/64</td>
</tr>
<tr>
<td>S2</td>
<td>$4_x$</td>
<td>$10_x$</td>
<td>10/64</td>
</tr>
<tr>
<td></td>
<td>$4_x$</td>
<td>$12_x$</td>
<td>10/64</td>
</tr>
<tr>
<td></td>
<td>$8_x$</td>
<td>$9_x$</td>
<td>10/64</td>
</tr>
<tr>
<td></td>
<td>$8_x$</td>
<td>$10_x$</td>
<td>16/64</td>
</tr>
<tr>
<td></td>
<td>$12_x$</td>
<td>$5_x$</td>
<td>14/64</td>
</tr>
<tr>
<td>S3</td>
<td>$4_x$</td>
<td>$9_x$</td>
<td>12/64</td>
</tr>
<tr>
<td></td>
<td>$8_x$</td>
<td>$3_x$</td>
<td>10/64</td>
</tr>
<tr>
<td></td>
<td>$12_x$</td>
<td>$5_x$</td>
<td>12/64</td>
</tr>
<tr>
<td></td>
<td>$12_x$</td>
<td>$6_x$</td>
<td>12/64</td>
</tr>
<tr>
<td>S4</td>
<td>$4_x$</td>
<td>$6_x$</td>
<td>12/64</td>
</tr>
<tr>
<td></td>
<td>$4_x$</td>
<td>$9_x$</td>
<td>12/64</td>
</tr>
</tbody>
</table>

### Boîte-S

<table>
<thead>
<tr>
<th>Boîte-S</th>
<th>$\delta_{in}$</th>
<th>$\delta_{out}$</th>
<th>Proba</th>
</tr>
</thead>
<tbody>
<tr>
<td>S5</td>
<td>$4_x$</td>
<td>$6_x$</td>
<td>10/64</td>
</tr>
<tr>
<td></td>
<td>$8_x$</td>
<td>$6_x$</td>
<td>10/64</td>
</tr>
<tr>
<td></td>
<td>$8_x$</td>
<td>$10_x$</td>
<td>10/64</td>
</tr>
<tr>
<td></td>
<td>$12_x$</td>
<td>$3_x$</td>
<td>10/64</td>
</tr>
<tr>
<td></td>
<td>$12_x$</td>
<td>$6_x$</td>
<td>10/64</td>
</tr>
<tr>
<td></td>
<td>$12_x$</td>
<td>$10_x$</td>
<td>10/64</td>
</tr>
<tr>
<td>S6</td>
<td>$8_x$</td>
<td>$6_x$</td>
<td>16/64</td>
</tr>
<tr>
<td></td>
<td>$12_x$</td>
<td>$3_x$</td>
<td>12/64</td>
</tr>
<tr>
<td></td>
<td>$12_x$</td>
<td>$5_x$</td>
<td>10/64</td>
</tr>
<tr>
<td>S7</td>
<td>$8_x$</td>
<td>$10_x$</td>
<td>12/64</td>
</tr>
<tr>
<td></td>
<td>$12_x$</td>
<td>$12_x$</td>
<td>14/64</td>
</tr>
<tr>
<td>S8</td>
<td>$4_x$</td>
<td>$12_x$</td>
<td>10/64</td>
</tr>
<tr>
<td></td>
<td>$12_x$</td>
<td>$5_x$</td>
<td>10/64</td>
</tr>
<tr>
<td></td>
<td>$12_x$</td>
<td>$6_x$</td>
<td>10/64</td>
</tr>
</tbody>
</table>
Introduction

Algebraic cryptanalysis of DES using Minisat

Algebraic-differential cryptanalysis of DES

Results on six, seven and eight rounds

\[
\begin{align*}
\Delta_0^L &= 00 \ 20 \ 20 \ 00 & \Delta_0^R &= 00 \ 00 \ 06 \ 00 \quad \text{with probability } \frac{12}{64} \\
00 \ 20 \ 20 \ 00 &\xrightarrow{F_{k_1}} 00 \ 00 \ 06 \ 00 \\
\Delta_1^L &= 00 \ 00 \ 06 \ 00 & \Delta_1^R &= 00 \ 00 \ 00 \ 00 \quad \text{with probability } 1 \\
00 \ 00 \ 00 \ 00 &\xrightarrow{F_{k_2}} 00 \ 00 \ 00 \ 00 \\
\Delta_2^L &= 00 \ 00 \ 00 \ 00 & \Delta_2^R &= 00 \ 00 \ 06 \ 00 \quad \text{with probability } \frac{12}{64} \\
00 \ 20 \ 20 \ 00 &\xrightarrow{F_{k_3}} 00 \ 00 \ 06 \ 00 \\
\Delta_3^L &= 00 \ 00 \ 06 \ 00 & \Delta_3^R &= 00 \ 20 \ 20 \ 00 \quad \text{with probability } 100/64^2 \\
04 \ 04 \ 01 \ 80 &\xrightarrow{F_{k_4}} 00 \ 20 \ 20 \ 00 \\
\Delta_4^L &= 00 \ 20 \ 20 \ 00 & \Delta_4^R &= 04 \ 04 \ 07 \ 80 \quad \text{with probability } 1 \\
04 \ 04 \ 07 \ 80 &\xrightarrow{F_{k_5}} 04 \ 04 \ 07 \ 80 \\
\Delta_5^L &= 04 \ 04 \ 07 \ 80 & \Delta_5^R &= \%\%\%0\%0\%0\%0\%0\%0\%0\%0\%0\%0\%0\%0\%0\%0
Experimental results

7 rounds cryptanalysis

- 2000 chosen plaintexts
- 3 hours

Not so much results on 7 rounds in the literature.
Eight rounds

We have found a 5-round truncated differential characteristic with probability $1/5800$. 
+ partial exhaustive search over 8 bits of the key.

8 rounds cryptanalysis

- 11600 chosen plaintexts and $2^{25}$ seconds.
### Summary

<table>
<thead>
<tr>
<th>Rounds</th>
<th>Cryptanalysis</th>
<th>Nb of plaintexts</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>diff (Biham, Shamir)</td>
<td>240 (chosen)</td>
<td>0.3 s</td>
</tr>
<tr>
<td></td>
<td>diff (Knudsen)</td>
<td>46 (chosen)</td>
<td>&lt;10 s</td>
</tr>
<tr>
<td></td>
<td>alg (Courtois, Bard)</td>
<td>1 (known)</td>
<td>$2^{25}$ s</td>
</tr>
<tr>
<td></td>
<td><strong>diff + alg</strong></td>
<td><strong>32 (chosen)</strong></td>
<td><strong>3000 s</strong></td>
</tr>
<tr>
<td></td>
<td><strong>diff + alg</strong></td>
<td><strong>22 (chosen)</strong></td>
<td>&lt;10 h</td>
</tr>
<tr>
<td>7</td>
<td><strong>diff + alg</strong></td>
<td><strong>2000 (chosen)</strong></td>
<td><strong>10000 s</strong></td>
</tr>
<tr>
<td>8</td>
<td>diff (Biham, Shamir)</td>
<td>50000 (chosen)</td>
<td>100 s</td>
</tr>
<tr>
<td></td>
<td>lin (Matsui)</td>
<td>$2^{20}$ (known)</td>
<td>40 s</td>
</tr>
<tr>
<td></td>
<td><strong>diff + alg</strong></td>
<td><strong>11500 (chosen)</strong></td>
<td><strong>$2^{25}$ s</strong></td>
</tr>
<tr>
<td></td>
<td>diff + lin (Hellman, Langford)</td>
<td>512 (chosen)</td>
<td>few seconds</td>
</tr>
</tbody>
</table>
Conclusion

- Use of statistical methods in algebraic cryptanalysis.
  → New attacks on 6, 7 and 8 rounds of DES using dedicated characteristics.
- Tradeoff plaintexts/time.
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Related work (Cryptanalysis of Present)

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Future work

- Extension of this attack for more rounds?
- Algebraic-differential cryptanalysis of DES with Gröbner Bases?
- Other cryptosystems?
- Other statistical tools (differential-linear cryptanalysis, . . .)?